Two Natural Θ_2^p-Complete Problems

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Two natural problems for which previously only NP-hardness or coNP-hardness lower bounds were known are shown to be complete for Θ_2^p (a.k.a. $\mathrm{P}_{||}^{\mathrm{NP}}$ or $\mathrm{P}^{\mathrm{NP}[\log]}$), the class of sets solvable via parallel access to NP.

In 1876, Lewis Carroll proposed an election system in which the winner is the candidate who with the fewest changes in voters' preferences becomes a Condorcet winner—a candidate who beats all other candidates in pairwise majority-rule elections. Bartholdi, Tovey, and Trick [BTT89] provided a lower bound—NP-hardness—on the computational complexity of determining the election winner in Carroll's system. We optimally improve their lower bound: Determining the winner in Carroll's system is Θ_2^p-complete. It is by far the most natural complete problem for this class. It follows from our result that determining the winner in Carroll's elections is not NP-complete unless the polynomial hierarchy collapses. This is joint work with Edith and Lane A. Hemaspaandra ([HHR97a], see also the survey [HHR97b]).

Bodlaender, Thilikos, and Yamazaki [BTY97] investigate the computational complexity of the problem of whether the Minimum Degree Greedy Algorithm can approximate the size of a maximum independent set of the input graph within a constant factor of r, for fixed rational $r \geq 1$. They denote this problem by $\mathcal{S}_r$ and prove that for each rational $r \geq 1$, $\mathcal{S}_r$ is coNP-hard. They also provide a P^{NP} upper bound of $\mathcal{S}_r$, leaving open the question of whether this gap between the upper and the lower bound of $\mathcal{S}_r$ can be closed. For the special case of $r = 1$, they show that $\mathcal{S}_1$ even is DP-hard (where DP denotes the class of sets that can be represented as the difference of two NP sets), again leaving open the question of whether $\mathcal{S}_1$ can be shown to be complete for DP or some larger class such as P^{NP}. We completely solve all the questions left open by Bodlaender et al. in [BTY97]. Our main result is that for each rational $r \geq 1$, $\mathcal{S}_r$ is complete for Θ_2^p. This is joint work with Edith Hemaspaandra [HR97].

Literatur

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