Padding and the Expressive Power of Existential Second-Order Logics

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Padding techniques are well-known from Computational Complexity Theory. Here, an analogous concept is considered in the context of existential second-order logics. Informally, a graph H is a *padded version of a graph* G, if H consists of an isomorphic copy of G and some isolated vertices. A set $\mathcal{A}$ of graphs is called *weakly expressible* by a formula φ *in the presence of padding*, if φ is able to distinguish between (sufficiently) padded versions of graphs from $\mathcal{A}$ and padded versions of graphs that are not in $\mathcal{A}$.

From results of Lynch it can be easily concluded that (essentially) every **NP**-set of graphs is weakly expressible by an existential monadic second-order ($\mathrm{m}\Sigma_1^1$) formula with polynomial padding and built-in addition.

In particular,

> **NP**$\neq$**coNP** if and only if there is a **coNP**-set of graphs that is not weakly expressible by a $\mathrm{m}\Sigma_1^1$-formula in the presence of addition, even if polynomial padding is allowed.

In some sense, this implies that $\mathrm{m}\Sigma_1^1$ is well suited to investigate the **NP** vs. **coNP** question.

In the talk, the following results will be presented:

- In the above statement, addition can be replaced by two unary functions, by built-in relations of degree $O(n^\epsilon)$, for every $\epsilon > 0$, and by built-in relations with $\leq (1+\epsilon)n$ edges, respectively;
- On the other hand, $\mathrm{m}\Sigma_1^1$ with built-in relations of degree $n^{o(1)}$ or with $n + n^{o(n)}$ edges is weak, in the sense that not every **P**-set of graphs is weakly expressible with polynomial padding in this logic; and
- $\mathrm{m}\Sigma_1^1$ with a built-in linear order or built-in coloured trees is very weak, in the sense that they are weak and padding does not help at all.
- Corresponding results hold for several sublogics of binary Σ_1^1.