On a Generic Uncertainty Model for Position Information

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Abstract. Position information of moving as well as stationary objects is generally subject to uncertainties due to inherent measuring errors of positioning technologies, explicit tolerances of position update protocols, and approximations by interpolation algorithms. There exist a variety of approaches for specifying these uncertainties by mathematical uncertainty models such as tolerance regions or the Dilution of Precision (DOP) values of GPS. In this paper we propose a principled generic uncertainty model that integrates the different approaches and derive a comprehensive query interface for processing spatial queries on uncertain position information of different sources based on this model. Finally, we show how to implement our approach with prevalent existing uncertainty models.

1 Introduction

Position information on moving objects such as mobile devices, vehicles, and users as well as stationary objects such as buildings, rooms, and furnishings is an important kind of context information for context-aware applications. The authors of [1] and [2] even refer to the locations of objects as *primary context*.

Position information is generally subject to uncertainties at every stage of processing: Already position information acquired by positioning sensors such as GPS receivers only approximates the actual position of the respective sensor or object due to physical limitations and measurement errors of the sensing hardware. Update protocols for transmitting position information from sensors to remote databases or location services further degenerate the position information for the sake of reduced communication cost [3–5]. Interpolating in time between consecutive pairs of position records may result in further uncertainties, depending on the temporal density of the position information. Fusion of position information on the same phenomenon improves the accuracy but cannot eliminate uncertainties altogether.

Many context-aware applications must not neglect such uncertainties. For instance, navigating a blind person around obstacles [6] requires estimates for the uncertainty of the position information about the blind person as well as about the obstacles.

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Therefore, a variety of mathematical models for uncertainty of position information have been researched and proposed in the last decades, depending on the specifics and properties of the different technologies and algorithms. For instance, GPS receivers model the distance between sensed and actual position by normal distributions depending on measurement errors and satellite constellation [7]. The authors of [8, 9] model all possible positions in-between two given position records by intersecting two circles around the positions given in the records, resulting in a lense-shaped area.

Based on these findings, different *uncertainty-aware* interfaces for accessing and querying position information have been proposed for the different system components such as positioning sensors, update protocol endpoints, moving objects databases, and location services.

With the advent of large-scale context-aware systems such as the Nexus platform [10], applications more and more rely on position information from many different sources. Therefore independent and technology-specific uncertainty models and query interfaces are not sufficient but a generic approach allowing for homogeneous and uncertainty-aware access to position information is required.

Regarding such an approach, we can again distinguish between a generic, mathematically principled model for uncertain position information and a suitable, extended query interface based on this generic uncertainty model. The requirements for the generic uncertainty model are as follows:

- *Expressiveness and generality:* The generic uncertainty model has to be fully compatible with all existing specific uncertainty models for position information of the different positioning sensors, update protocols, and fusion and interpolation algorithms and reflect them with minimal loss of information.
- *Directness:* For simplicity and for minimizing the computational and storage overhead in implementations, the generic uncertainty model has to represent the uncertainty of position information in a self-evident way corresponding to the specific uncertainty models.

The resulting basic requirements for the extended, uncertainty-aware query interface are the following ones:

- *Immediacy and comprehensiveness:* The query interface should immediately build upon the generic uncertainty model to minimize computational effort and exploit all information provided by the uncertainty model.
- *Generality:* The query interface has to provide all prevalent spatial query types for position information such as position query, range query, and next-neighbor query, cf. [11].

In this paper we propose a generic uncertainty model based on partial spatial distribution functions and a corresponding extended query interface supporting five prevalent query types satisfying the above requirements. Our approach is suitable for all (uncertain) point-shaped position information in the two-dimensional space.

In detail, we present the following contributions: In Section 2, we survey existing, specific uncertainty models for position information. In Section 3, we show that they can be classified into three fundamental types and that they all base on partial spatial distribution functions and then derive our generic uncertainty model. Based on this finding we present an extended query interface for uncertain position information in Section 4 and show how to implement this interface for different specific uncertainty models. Section 5 discusses related work, before we present our conclusions and outlook in Section 6.

2 Survey of Specific Uncertainty Models

In this section, we survey the existing uncertainty models for position information. Most of these models only consider two-dimensional positions. Height information of indoor positioning systems is often reduced to information about the floor and, in case of outdoor systems such as GPS, the height information is handled separately. Therefore, we restrict this survey to two-dimensional positions.

2.1 Uncertainty Models of Positioning Systems

Positioning systems use different techniques like triangulation/-lateration, scene analysis, or proximity sensing to determine positions [12]. Different positioning techniques not only yield different scales of accuracy—from millimeters to hundreds of meters—but also result in different uncertainty models.

Positioning systems based on trilateration mostly model the position sensed at time t and denoted by s_t as a normal distribution. This particularly applies to global navigation satellite systems such as GPS [7] and ultrasonic-based positioning systems such as Cricket [13]. For instance, according to [7] the standard deviation σ of a two-dimensional position determined by GPS can be calculated based on the User Equivalent Range Error σ_{UERE} and the Horizontal Dilution of Precision, HDOP, as

$$\sigma = \text{HDOP} \cdot \sigma_{\text{UERE}} \tag{1}$$

Other systems—e.g., the WiFi positioning system presented in [14]—only give a center point and several percentile values around that point, i.e. s_t consists of several concentric circles expressing the probability that the actual position, denoted by a_t, lies within a given circle. In beaconing systems, such as Active Badge [15], the sensed position may only specify an area such as a room—i.e. a_t is known to be in that area but without any further distribution information. The same applies to the Smart Floor positioning systems using pressure-sensitive tiles [16] and positioning using passive RFID tags [17].

2.2 Modeling of Uncertainty in Update Protocols

Update protocols introduce further uncertainties to position information and lead to new uncertainty models. Dead reckoning protocols trade accuracy off against communication cost for efficient transmission of position information from a remote positioning sensor to stationary components managing the current position [3, 4]. Remote trajectory simplification algorithms additionally consider the costs for storing the current and past positions, i.e. the whole trajectory [18, 5]. For all these approaches, a position s_t is modeled by a center point and a distance value for the maximum distance from that point. However, the distribution within the resulting circle is undefined.

2.3 Uncertainty Models for Fusion and Interpolation

Fusion algorithms improve the accuracy of a sensed position from different sensor data on the same phenomenon. Multi-Area Probability-based Positioning by Predicates [19] describes s_t by a number of polygons with probability values taking into account even multiple predicates on the position. For fusion of arbitrary probability density functions different Bayes filter implementations are applicable, possibly discretizing the plane using a grid [20].

Complex, uncertainty-aware interpolation algorithms allow for deriving positions at times in-between two sensing operations taking into account the temporal discretization introduced by sensing. The authors of [8, 9] show how to restrict the position s_t at such a time to a lense-shaped area—the intersection of two circles—by means of the maximum speed between the sensing operations.

3 Mathematical Generalization of Uncertainty Models

The diversity of these specific models makes it difficult to incorporate uncertain position information from different sources in applications. We show in the next section, however, that all these different models can be reduced to a common mathematical model for uncertain position information. After that, we propose a consistent interface for applications that need to access uncertain position data from different specific models. This interface is based on our common mathematical model.

3.1 Classification of Uncertainty Models

Although a large number of different specific models for uncertain position information exists, we can classify them into three major types as illustrated in Figure 1:

1. *pdf-based models:* These models use complete probability density functions to describe the uncertainties of positions. Hence, with such a model, a position at time t is described by a two-dimensional probability density function $s_t : \mathbb{R}^2 \to [0, \infty)$.
   Amongst others, pdf-based models are used for specifying the uncertainty of trilateration-based positioning systems such as GPS.

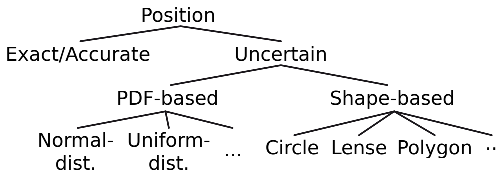

Fig. 1. Taxonomy of major classes of the existing uncertainty models.

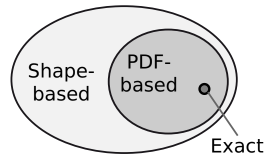

Fig. 2. Mathematical generality of the major uncertainty models.

2. *Shape-based models:* Models of this class describe positions by geometric shapes. These shapes have associated probability values, however, in contrast to the pdf-based models, the approaches make no claims about the probability distribution within a shape.
   Hence, a position at time t is a set $s_t = \{(A_1, p_1), \ldots, (A_n, p_n)\}$ where $p_j \in [0, 1]$ and $A_j \subseteq \mathbb{R}^2$ are geometric shapes such as polygons or circles.
   Shape-based models are used, for example, for position information from infrared beacons, RFID tags, or interpolation with the intersection of circles.
3. *Accurate model:* For completeness, we also include the accurate model for specifying an exact position without uncertainty.
   Formally, a position is described by a single point, representing the actual position, i.e. $s_t = a_t$.

3.2 Generic Uncertainty Model

In terms of probability theory, all three classes of uncertainty models provide probabilistic information on the actual position they describe as they allow for mapping from one or more geometric shapes A to cumulative probabilities p. More precisely, they describe the position at time t by a (generally partial) function $s_t : \mathcal{P}(\mathbb{R}^2) \rightarrow [0, 1]$ with

$$s_t(A) = P[a_t \in A] = p \ . \tag{2}$$

We refer to such a function as partial spatial distribution function (psdf). Note that pdf-based models even allow for computing a mapping for all $A \in \mathcal{P}(\mathbb{R}^2)$ and thus can be treated as special, non-partial cases of psdf. Accurate positions given by the accurate model likewise are special cases of psdf, where $s_t(A) = 1$ if $a_t \in A$, and otherwise $s_t(A) = 0$.

Thus, regarding psdf, the three classes of uncertainty models can be nested according to their mathematical generality as illustrated in Figure 2.

It holds that $A_j \subseteq A_k$ implies $s_t(A_j) \leq s_t(A_k)$ as well as $s_t(\mathbb{R}^2) = 1$. Thus, given an arbitrary area A, a psdf s_t allows for deriving two estimates p_{lower} and p_{upper} with $0 \leq p_{\text{lower}} \leq s_t(A) \leq p_{\text{upper}} \leq 1$ for the position of the corresponding object at time t.

The three major classes of uncertainty models and their common basis in terms of probability theory is an important finding and composes the generic uncertainty model for the extended query interface proposed in the next section.

As the generic uncertainty model includes all classes of specific uncertainty models, it satisfies the requirements *expressiveness* and *generality* given in Section 1. Furthermore, it satisfies *directness* as it immediately bases on shape-based models, the most general class of uncertainty models in mathematical terms.

4 Uncertainty-aware Query Interface

In this section we present an extended, uncertainty-aware query interface for position information based on the above finding of a generic uncertainty model. Therefore, the query interface can be implemented for every existing uncertainty model and thus allows for uncertainty-aware processing of position information from different, heterogeneous sources.

Next, we discuss the extended, uncertainty-aware versions of prevalent spatial query types for position information and thus show that the query interface meets the requirement *generality* given in Section 1. Then, we describe a number of examples how to implement the query types and thereby the query interface for different specific uncertainty models, which shows that the query interface also satisfies the requirements *immediacy* and *comprehensiveness*.

4.1 Extended Query Types

In the following, we consider an arbitrary set of moving or stationary objects $\{O_1, \ldots, O_n\}$. Though most entities of the real-world have a certain extent, we only consider point objects. For any given object, one can always define an anchor point and thus reduce its position to this point. We denote the position of object O_i at time t by $s_{i,t}$. All queries have two parameters O_i and t in common, specifying the queried object and the queried time, respectively.

Besides the uncertain position information, we argue that the providers must define most likely point positions for each time and object they manage. This *defined point* $c_{i,t}$ for an object O_i at time t may be either modeled explicitly or computed on the fly from the uncertain position information of O_i. Note that this point is naturally given with most existing uncertainty models such as normal distributions or circular shapes.

The defined point $c_{i,t}$ of O_i at time t serves to define an unambiguous mapping $\bar{c}_{i,t} : [0, 1] \to \mathcal{P}(\mathbb{R}^2)$ from each cumulative probability p to the circular area $\bar{c}_{i,t}(p) = A^{\mathrm{C}}$ with center $c_{i,t}$ and minimum radius such that $s_{i,t}(A^{\mathrm{C}}) \geq p$. This is needed, as the inverse $s_{i,t}^{-1}(p)$ is generally ambiguous. For instance consider the 2D normal distribution illustrated in Figure 4: The left half, the right half, and the inner circle are three examples of areas with $s_{i,t}(A) = 0.5$.

Where applicable, the circular areas $\bar{c}_{i,t}(p)$ are clipped to $A^1_{i,t}$, the smallest area with $s_{i,t}(A^1_{i,t}) = 1$, which always is unambiguous but may be equal to $\mathbb{R}^2$.

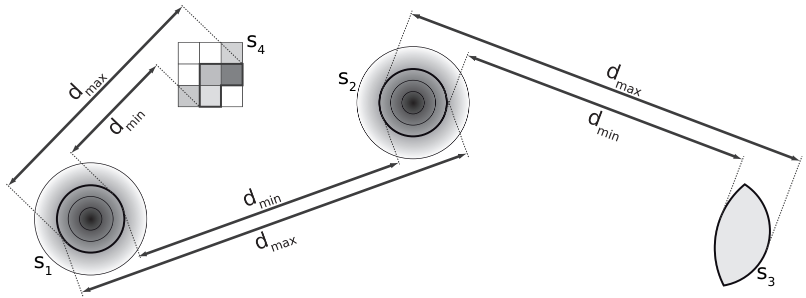

Fig. 3. Distance query evaluation.

Position Query Besides O_i and t, the position query takes a parameter $p \in [0, 1]$ and returns the smallest area $A = \bar{c}_{i,t}(p) \cap A^1_{i,t}$ such that $s_{i,t}(A) \geq p$:

$$\text{Position Query:} \quad \text{PQ}\,(O_i, t, p) \rightarrow (A, \bar{c}_{i,t}(p), s_{i,t}(A)) \tag{3}$$

Moreover, it returns $\bar{c}_{i,t}(p)$ and the probability value $s_{i,t}(A)$.

Inside and Range Query To test whether an object is within an area A with a probability of at least $p_{\text{true}} > p_{\text{false}}$, the inside query is defined as:

$$\text{Inside Query:} \quad \text{IQ}\,(O_i, t, A, p_{\text{true}}, p_{\text{false}}) \rightarrow (\{\texttt{true}, \texttt{maybe}, \texttt{false}\}) \tag{4}$$

With the estimates for p_{lower} and p_{upper} from Section 3, the inside query returns `true` iff $p_{\text{lower}} \geq p_{\text{true}}$ and `false` iff $p_{\text{upper}} \leq p_{\text{false}}$. In all other cases, the uncertain position obviously overlaps the area A as well as its inverse $\mathbb{R}^2 \setminus A$ and the query returns `maybe`.

The range query can be implemented easily by inside queries on the set of queried objects.

Distance and Nearest-Neighbor Query The distance query returns an upper and lower bound for the distance between two objects with a minimum probability of p by computing the minimum and maximum distances $d_{\min}$ and $d_{\max}$ between the two shapes $\bar{c}_{i,t}(p) \cap A^1_{i,t}$ and $\bar{c}_{j,t}(p) \cap A^1_{j,t}$.

$$\text{Distance Query:} \quad \text{DQ}\,(O_i, O_j, t, p) \rightarrow (d_{\min}, d_{\max}) \tag{5}$$

Figure 3 illustrates several examples of how $d_{\min}$ and $d_{\max}$ are computed. In Section 4.2, these examples are discussed in detail.

The nearest-neighbor query uses these distance bounds to derive the set of objects that may be closest to a given object O_i. Given O_i and a probability value p, it computes the pairwise distances between O_i and all other objects O_j $(i \neq j)$ as described above and determines the maximum distance $\hat{d}$ for O_i's

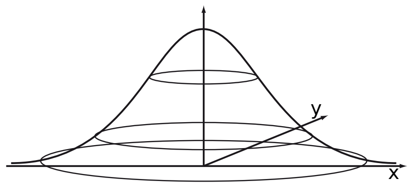

Fig. 4. 2D normal distribution.

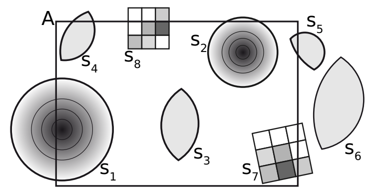

Fig. 5. Range query evaluation.

nearest neighbor as $\hat{d} = \min(d_{\max})$ for all $O_j \neq O_i$. Then, it returns the set of objects with their distance bounds that may be closer to O_i than $\hat{d}$:

$$\text{Nearest-Neighbor Query:} \quad \text{NNQ}\,(O_i, t, p) \rightarrow (O_j, d_{\min}, d_{\max})^* \ \text{ where } d_{\min} \leq \hat{d}$$

Thus, depending on p, the result either contains only objects that are likely to be nearest neighbors or also objects with a low probability of being nearest neighbor.

4.2 Implementing the Query Interface

In the following, we exemplary discuss how to implement the proposed query interface for three specific uncertainty models. For actual implementations different spatial data models such as the simple feature types of the Open Geospatial Consortium (OGC) are feasible[1].

As a first example, we consider the uncertainty model of GPS [7] based on normal distributions. Then, we discuss the lense-based uncertainty model of the interpolation algorithm in [8, 9]. Finally, we show how to map a grid-based uncertainty model [20] to the extended query interface. For all of these models, we show how to implement *PQ*, *IQ*, and *DQ*. We leave out *RQ* and *NNQ* since these are straight-forward extensions of *IQ* and *DQ*.

GPS Uncertainty Model based on Normal Distribution A GPS position is given by longitude, latitude, and the HDOP value specifying a 2D normal distribution. Longitude and latitude can be directly used as defined point $c_{i,t}$ for the generic model. As already described in Equation 1, the HDOP value is multiplied with the device-specific User Equivalent Range Error, σ_{UERE}, to derive the standard deviation σ of the normal distribution [7].

As an example consider a GPS sensor with accuracy $\sigma_{\text{UERE}} = 5\,\text{m}$ reporting $(lat, lon, HDOP)$ as $(48°47'N, 9°11'O, 1.5)$. In this case, a PQ with $p = 0.75$ returns a circle A^{C} that is centered at $(48°47'N, 9°11'O)$ with radius $r = 8.84\,\text{m}$ by solving the circular integral over the density function $f_{N^2(0,\sigma^2)}(x, y)$ of the

[1] Depending on the data model, curves (e.g., of circles or lenses) have to be approximated by polygons at a suitable level of granularity.

two-dimensional normal distribution with $\sigma = \text{HDOP} \cdot \sigma_{\text{UERE}} = 1.5 \cdot 5\,\text{m} = 7.5\,\text{m}$ for r, i.e. by solving

$$p = \int_{\sqrt{x^2+y^2} \leq r} f_{N^2(0,\sigma^2)}(x,y)\,\mathrm{d}(x,y)\ . \tag{6}$$

Figure 5 shows a queried range A and the positions of eight objects. s_1 and s_2 are GPS positions where an IQ can be unambigously evaluated by integrating $f_{N^2(0,\sigma^2)}(x,y)$ over A. For $p_{\text{true}} = 0.8$ and $p_{\text{false}} = 0.2$, IQ returns `maybe` for s_1 and `true` for s_2.

Figure 3 shows several positions of objects and the upper and lower bound for the distances between pairs of these positions. In case of a DQ on two GPS positions s_1 and s_2 with $p = 0.75$, A^{C} is computed for each of these positions according to the explanations for the PQ. The lower bound $d_{\min}$ for the distance between the positions is then computed as the minimal distance between the resulting circles. Similarly, the upper bound $d_{\max}$ is computed as the maximal distance between these circles.

Lense-based Uncertainty Model For interpolation with lenses [8, 9] (cf. Section 2), we consider two consecutive position fixes of an object O_i at times $t_1 = 0\,\text{s}$ and $t_2 = 100\,\text{s}$ in the Euclidean plane with $s_{i,t_1} = (0\,\text{m}, 0\,\text{m})$ and $s_{i,t_2} = (100\,\text{m}, 0\,\text{m})$. In addition, we assume the maximum speed of the object is known to be $1.5\,\text{m/s}$. The position query PQ for time $t = 50\,\text{s}$ returns the intersection of the circles centered at s_{i,t_1} and s_{i,t_2} with radius $r = 1.5\,\text{m/s}{\cdot}50\,\text{s} = 75\,\text{m}$ for any queried p. Note that the probability $s_{i,t}(A)$ given in the query result always is 1.0.

Also note that any point within the lense can be chosen as defined point $c_{i,t}$ without affecting the result $A = \bar{c}_{i,t}(p) \cap A^1_{i,t}$ of the PQ (cf. Equation 3). An obvious choice for $c_{i,t}$ is the linear interpolation between s_{i,t_1} and s_{i,t_2}.

For the queried range A given in Figure 5, the IQ returns `true` for position information s_3, `false` for s_6, and `maybe` for s_4 and s_5 for any value of p specified in the queries.

Figure 3 shows an example for a DQ involving a lense-based position s_3. To evaluate the DQ between s_3 and the GPS position s_2 with $p = 0.75$, a PQ on s_3 is processed, which results in a lense shape. Then, the lower bound $d_{\min}$ for the distance between the positions is computed as the minimal distance between the lense shape of s_3 and A^{C} of s_2. Similarly, the upper bound $d_{\max}$ is computed as the maximal distance.

Grid-based Uncertainty Model Consider a grid-based [20] position that is given by a set of tuples (x_j, y_j, p_j) where x_j, y_j are cell coordinates and p_j is the corresponding probability. Thus, the grid-based position is given by a set of disjoint quadratic shapes with associated probabilities. As defined point $c_{i,t}$ for the generic uncertainty model, a couple of alternatives are conceivable: First, the center of the cell with highest probability p_j is chosen. Second, the defined point is selected as the centroid.

As an example consider a grid-based position defined by

$$s_{i,t} = (1,1,0.15),(2,1,0.05),(2,2,0.2),(3,2,0.5),(3,3,0.1)\ .$$

We compute the centroid (x_c, y_c) of this position as defined point by taking the weighted sum of cell indices over each dimension:

$$x_c = 1 \cdot 0.15 + 2 \cdot (0.05 + 0.2) + 3 \cdot (0.5 + 0.1) = 2.45$$
$$y_c = 1 \cdot (0.15 + 0.05) + 2 \cdot (0.2 + 0.5) + 3 \cdot 0.1 = 1.9$$

A PQ with $p = 0.75$ is evaluated by selecting the cells closest to the centroid until the aggregated probability of the cell equals or exceeds 0.75. In this example the polygon enclosing the cells $(2,1,0.05),(2,2,0.2),(3,2,0.5)$ is returned. For $s_{i,t}(A)$, a value of 0.75 is returned, since the sum of these cells' probabilities equals 0.75.

For the queried range A in Figure 5, the IQ for the grid-based position information s_7 can be evaluated unambiguously since the range A is aligned to its grid. As the grid of position s_8 is not aligned to the range A, p_{upper} and p_{lower} differ. p_{lower} is the sum of probabilities of the cells that are covered by range A. In contrast, p_{upper} is evaluated as the sum of probabilities of cells that overlap with A.

Figure 3 shows an example for a DQ involving a grid-based position s_4. The processing of the DQ between s_4 and the GPS position s_1 with $p = 0.75$ is based on the result of a PQ on s_4. With the grid-based uncertainty model, a PQ results in a polygonal area possibly consisting of multiple unconnected parts. The lower bound $d_{\min}$ for the distance is computed as the minimal distance between A^C of s_1 and the nearest part of the area returned by the PQ. The upper bound $d_{\max}$ is computed as the maximal distance to the most distant part.

5 Related Work

The proposed query interface for uncertain position information and its generic uncertainty model relates to two fields: Models for uncertain spatial data in general and specific approaches for uncertain position information of moving objects.

Pauly and Schneider [21] classify the former into models based on rough sets like the Egg-Yolk approach [22] and models based on fuzzy sets like the fuzzy spatial data types proposed in [23]. The models particularly define topological predicates for vague spatial regions but do not aim at a generic model integrating the variety of existing uncertainty models.

A variety of algorithms for processing range and next-neighbor queries on uncertainty position information have been proposed in recent years, e.g., [9, 24, 25]. Most approaches model uncertain positions as circular areas which can be mapped to the proposed generic model. Moreover, they use compatible semantics for query results such as the MAY/MUST semantics for the containment in queried regions proposed in [25].

Existing approaches for fusion of position sensor data—particularly Bayesian filtering [20] and inferring from location predicates [19]—are also covered by the generic uncertainty model as discussed in the previous sections.

6 Conclusions and Outlook

In this paper we discussed the need for a generic uncertainty model for position information in large-scale context-aware systems and formulated the requirements for a suitable model and uncertainty-aware query interface.

We surveyed and classified the variety of existing technology-specific uncertainty models and showed that they all can be considered as partial spatial distribution functions (psdf) with respect to their mathematical generality.

Based on this finding, we proposed an extended query interface for position information by extending common query types with information on the position uncertainty. Furthermore, we discussed how to implement these types for certain prevalent uncertainty models. These examples show that the proposed query interface can provide homogeneous access to uncertain position information from different sources and sensors and that the proposed approach meets the various requirements formulated in Section 1.

Although we only discussed position information in this paper, the mathematical approach can be extended easily to scalar data types (e.g., temperature and velocity) as well as data with three or more dimensions. Of course, the query interface has to be adapted to the relevant query types for these data types.

Currently, we only consider firm boundaries for the queried ranges with inside and range queries. In the future, we will expand our approach to also support ranges with uncertainties.

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