Integration of Communication Networks and Control Systems Using a Slotted Transmission Classification Model

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***Abstract*—In this paper, we present a communication abstraction for Networked Control Systems that is characterized by a slotted transmission classification model. We discuss, how such a model can be implemented over local area networks by using IEEE Time Sensitive Networking methods. Furthermore, it is shown how asymptotic stability can be analyzed for linear systems that communicate over such a network. Based on the stability result, a controller design procedure is derived that takes the information captured in the network model into account. Further topics and related open problems that are implicated by the proposed model are briefly discussed as an outlook.**

***Index Terms*—Cyber-Physical Networks, Network Architectures and Protocols, Control Design and Architectures, Time-Sensitive Networking, Stability.**

I. Introduction

Control systems in which feedback loops are closed over a packet-switched network are known as Networked Control Systems (NCS) [1]. NCS enable an economical and flexible deployment of control applications such as, e.g., smart factories and autonomous traffic systems.

Traditionally, NCS have often been treated solely from the perspective of control theory. A consequence of such approaches that perform a strict separation between the control theoretic investigation and the network implementation, is that only limited knowledge about the communication network is exploited for efficient resource utilization. However, due to the expected increase of the number of networked applications in cyber-physical networking scenarios, this will be an important issue in the future. This underlines the demand for an integrated approach to communication and control for the design and analysis of NCS.

One possibility for enforcing integrated approaches is to define a common communication abstraction. Such an abstraction should capture relevant information about the network, such that this information can be exploited for controller design. At the same time, the abstraction should form the basis for an implementation of communication networks that satisfy the agreed upon properties by design. One recent example, where such a common set of properties is used to facilitate the integrated design of NCS, is given by [2] using a wireless bus abstraction.

An alternative communication abstraction will be presented in this paper by a model that is based on a slotted transmission classification. The idea of differentiating the Quality of Service (QoS) according to different time slots has been successfully introduced for a CAN bus model in [3], where mandatory and optional control jobs are considered. In the model that we present here, this concept is extended to local area networks, specifically IEEE 802.3 Ethernet, by illustrating feasibility using *Time-Sensitive Networking* (TSN) methods, which have been recently standardized in IEEE 802.1Q.

The control theoretic motivation for such an abstraction is the potential to separate the concerns of guaranteeing stability with *deterministic transmissions* and increasing the control performance using *opportunistic transmissions*. The focus in this paper lies on the stability guarantees *independent* of any assumptions about opportunistic transmissions. The stability analysis for a given set of parameters of the communication abstraction is considered. Furthermore, the problem of designing a controller that takes information about the abstraction into account is considered, which follows the ideas of our results for weakly hard real-time systems [4].

The remainder of the paper is organized as follows. The considered network model is presented in Section II, while its implementation using TSN is discussed in Section III. In Section IV, the stability analysis and synthesis results are derived. An outlook is given together with the conclusion in Section V.

Notation: In the following, $\mathbb{N}$ denotes the set of positive integers and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. For symmetric matrices P, Q, the

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notation $P \succ Q$ is used to write that $P - Q$ is positive definite, while 0 and I denote zero and identity matrices of suitable dimensions. We denote the cardinality of a set S by $|S|$.

II. Slotted Transmission Classification Model

We begin by introducing a generic model for a set of NCS using a shared network, where we specifically model their transmission behaviour. Based on this, we propose a model for the time-slotted communication service described above.

Consider a set of N different NCS. Each NCS consists of a physical plant, which is the dynamical process to be controlled, a sensor, and an actuator. Sensors and actuators are logically colocated with their corresponding plants. However, sensor and actuator *nodes* are in general positioned at separate locations in the network, either topologically on different hosts, even if in close physical proximity, or geographically, if the plant represents a spatially distributed physical system.

Each plant is regulated by a controller, which we assume to be located either at the sensor or the actuator node. The sensor nodes send datagrams to the actuator nodes over a shared communication medium. Depending on the location of the controller, the payload of a datagram is either a measurement of the plant state generated by the sensor for the controller, or a control value generated by the controller for the actuator. We will specify a possible application model of control systems for one of these cases in Section IV-A. Moreover, datagrams are tagged at the source with a binary traffic classification (opportunistic or deterministic).

In the following, the superscript index i is used to distinguish between individual NCS in formulae where necessary. As just mentioned, we distinguish between deterministic (guaranteed) and opportunistic transmissions in order to separate the two criteria stability and performance enhancement to a certain degree. For this, we perform a time-based classification of datagrams into these two traffic classes.

We assume that there is an underlying grid of *timeslots* for all transmissions, which is determined by the sampling times t_k of the plants, following a common sampling period T_s, i.e., $t_{k+1} - t_k = T_s$ for all $k \in \mathbb{N}$. For each NCS i, we introduce two parameters s^i and d^i, where $0 \leq s^i < d^i$, which denote the phase shift and period of that system's deterministic transmissions, respectively. Thereby, all transmission times t_k of the deterministic transmissions of NCS i are characterized by

$$(k - s^i) \bmod d^i = 0, \tag{1}$$

and we denote the set of all NCS with a deterministic transmission at time t_k as $D_k = \{i \mid (1)\} \subseteq \{1, \ldots, N\}$. For deterministic slots, we assume that datagrams are marked accordingly, and

$$i \in D_k \implies \theta_k^i = 1, \tag{2}$$

as the corresponding transmissions are guaranteed. Here, θ_k^i indicates whether the datagram sent by NCS i at time t_k has been successfully received within the sampling interval

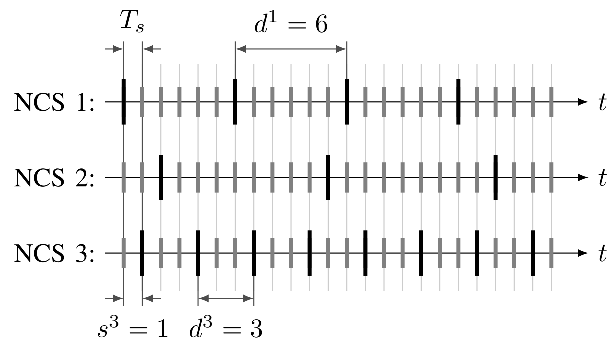

Fig. 1. Slotted transmission model illustrated for three NCS, each with deterministic slot period d^i and shift s^i. Underlying transmission grid determined by common sampling period T_s. *Deterministic* transmission slots indicated by black bars, *opportunistic* slots by shortened grey bars.

($\theta_k^i = 1$) or not ($\theta_k^i = 0$). All other transmission slots are opportunistic, and carry (in general) no *a priori* guarantees.

Deterministic transmissions are handled as (isolated) real-time traffic with minimal queueing delay. We assume that the network provides a certain capacity q_{det} for the number of total deterministic transmissions in one time slot, and therefore assume that $\left|D_k\right| \leq q_{\text{det}}$. This corresponds to a resource reservation in the network subject to the constraint that the end-to-end transfer delay for a burst of q_{det} NCS datagrams does not exceed the sampling period T_s. An illustration of a possible configuration with three NCS and $q_{\text{det}} = 1$ is shown in Fig. 1, where $(d^1, s^1) = (6, 0)$, $(d^2, s^2) = (9, 2)$, and $(d^3, s^3) = (3, 1)$. Note that q_{det} is not required to be constant, but can be taken as time-varying without loss of generality.

III. Scheduling of (Guaranteed) Real-time Transmission Slots in TSN

In this section, we discuss how time-triggered real-time transmission slots, in particular for deterministic transmissions, can be scheduled in IEEE 802.3 networks using standard TSN mechanisms.

A. Background

It is worth noting that the TSN standards define multiple scheduling mechanisms. However, the most important one to ensure low and deterministic network delay bounds and jitter is the so-called *Time-aware Shaper* specified in IEEE 802.1Qbv "Enhancements for Scheduled Traffic" [5]. This scheduler uses so-called *gates* to control when each of the first-in-first-out (FIFO) queues, associated with a particular egress port of a switch, can transmit buffered packets over that port at any given time, as shown in Fig. 2. A switch might implement up to eight queues for egress traffic per port. Each packet is tagged with a three-bit *Priority Code Point* (PCP), which is part of the Virtual LAN (VLAN) header, that is mapped to a corresponding queue where the forwarded frame is enqueued during forwarding. Buffered frames from a queue can only be transmitted over the corresponding port if the gate associated with this queue is open.

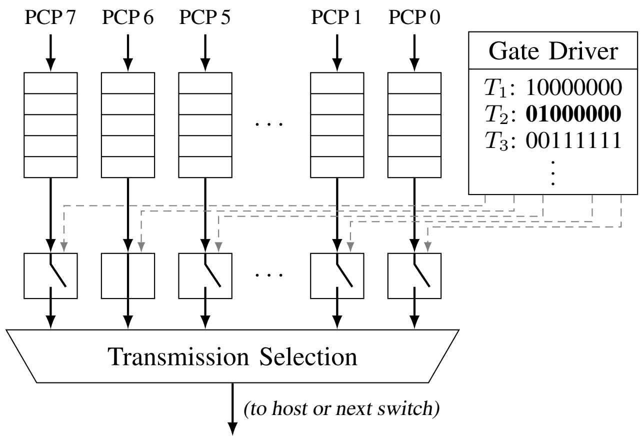

Fig. 2. Gating and transmission selection architecture for one port of a switch complying with IEEE 802.1Qbv. Packets traverse this queueing system from top to bottom. The figure shows a situation at some time $T_2 \leq t < T_3$, where only the gate for queue 6 is open.

The *Gate Control List* (GCL) defines the schedule for opening and closing the gates. To this end, the clocks of all switches must be synchronized using, e.g., the Precision Time Protocol (PTP, IEEE 1588) [6]. Each GCL entry contains a time stamps and associated bit vector indicating which gates are to be opened (1) or closed (0) at that time. The schedule is cyclic, i.e., it repeats in an endless loop after the last entry. The gate schedules of switches can be configured from a (logically) centralized network controller using the Simple Network Management (SNMP), NETCONF, or RESTCONF protocol.

It is important to realize that multiple gates might be open at the same time. In that case, another scheduling algorithm decides which of the queues with open gates is eligible for transmitting frames. For instance, the *Strict Priority* transmission selection algorithm as specified in IEEE 802.1Q can be used to this end using the PCP to define queue priorities, such that the next frame to be transmitted on a particular egrees port would always be taken from the head of the highest priority non-empty queue. Note, however, that the Time-aware Shaper always takes precedence over other scheduling mechanisms. In the scenario depicted in Fig. 2, only frames with a PCP value of 6 would be transmitted at times $t \in [T_2, T_3)$, even if the higher-priority queue 7 were not empty at the same time.

It is worth pointing out that the IEEE standards relating to TSN define several other interesting features such as Frame Preemption and additional traffic shaping and transmission selection policies. However, we limit our presentation to those mechanisms which illustrate the feasibility of our proposed service model for NCS.

B. End-to-end Service Guarantees

The standards described above allow us to establish dedicated time slots on each port in a local area network, where the packets belonging to a certain traffic class as defined by the PCP — or to a set of traffic classes subject to PCP precendence — have exclusive medium access (for the following hop). We can use this to provide service guarantees to a *flow* of packets between two end systems by ensuring that (a) each end system transmits packets (with the corresponding PCP) only at certain points in time according to a *host schedule*, (b) each switch along the flow's forwarding path is configured such that the gate for the flow's PCP on the appropriate egress port is open (with all higher-priority gates closed) when (or shortly after) a packet belonging to this flow arrives at the switch, and (c) the number of packets ahead of any of the flow's packets in the corresponding queues is limited appropriately, e.g., to fewer than can be transmitted in the period during which the gate is open.

Of course, the level of QoS that can be provided to time-sensitive flows using TSN methods depends on the degree of coordination between all host and gate schedules throughout the network. In principle, assuming perfect clock synchronization, any queueing delay can be eliminated for at least one flow. For multiple flows, the gate schedules have to be calculated such that the properties described above are satisfied for all flows.

C. Gate Schedule Calculation

While IEEE 802.1Qbv defines the time-triggered gating and traffic selection mechanisms, the calculation of appropriate gate schedules (possibly along with the corresponding host schedules) is out of the scope of the standard. Different algorithms for calculating the schedules have been proposed in the literature [7]–[9], typically resulting in complex constraint satisfaction and optimization problems to meet delay and jitter bounds, and optimize network utilization or similar metrics. For instance, [7] describe an Integer Linear Programming (ILP) formulation for calculating compact schedules (i.e., requiring a minimal number of GCL entries) for a set of flows with periodic host schedules with bounded delay. Without going into further detail about these existing scheduling approaches, we point out that they provide a service model that corresponds very well to the transmission schedules in our proposed communication service model. In the following section we discuss how the guarantees provided for deterministic transmissions can be exploited to reason about stability of the closed-loop NCS.

IV. Stability Analysis and Synthesis Using the Presented Network Model

We first present an exemplary application model. For the combination of this application model and the network model presented in Section II, we will then show how stability analysis and synthesis results can be derived for two different types of QoS guarantees for opportunistic transmissions.

A. Application Model

As an application model, we consider N, possibly heterogeneous, linear discrete-time control systems with sampling time T_s. Thus, for all control systems $i \in \{1, \dots, N\}$, the state $x_k^i := x^i(t_k) \in \mathbb{R}^{n^i}$ evolves according to the system dynamics

$$x_{k+1}^i = A^i x_k^i + B^i u_k^i, \quad \forall k \in \mathbb{N}_0, \tag{3}$$

with initial condition $x_0^i \in \mathbb{R}^{n^i}$ and input $u_k^i \in \mathbb{R}^{p^i}$ that is assumed to be zero initially $u_0^i = 0$ for all control systems. We assume that all pairs of matrices (A^i, B^i), where $A^i \in \mathbb{R}^{n^i \times n^i}$ and $B^i \in \mathbb{R}^{n^i \times p^i}$, are stabilizable.

Here, we consider the scenario of a controller that is located at the sensor. Thus, the payload of each datagram is a computed control value. This control value is computed using a static state feedback controller

$$v_k^i = K^i x_k^i, \quad \forall k \in \mathbb{N}_0, i \in \{1, \ldots, N\}. \tag{4}$$

The controller gains $K^i \in \mathbb{R}^{p^i \times n^i}$ for all control systems $i \in \{1, \ldots, N\}$ are either given and stability of the closed loop will be analyzed; or the gains are designed using a synthesis result.

As discussed in Section II, θ_k^i indicates whether the datagram sent by NCS i at time t_k, i.e., v_k^i, has been successfully received within one sampling interval ($\theta_k^i = 1$) or not ($\theta_k^i = 0$). If a datagram is received within the sampling interval, i.e., T_s seconds, it is buffered such that the induced delay is constant, i.e., v_k^i can either be applied at time t_{k+1} if $\theta_k^i = 1$ or it is treated as a lost packet and never used. For treating lost packets in control, there are two popular strategies, called *zero* and *hold* strategy. To simplify notation, we will only consider the hold strategy in the following, where the applied control value is held constant until a new control value arrives at the actuator. We refer for a general discussion of the two strategies to [10]. Thus, the applied control input of plant i at time t_k is given by

$$u_k^i = \theta_{k-1}^i v_{k-1}^i + (1 - \theta_{k-1}^i) u_{k-1}^i, \forall k \in \mathbb{N}, i \in \{1, \ldots, N\}. \tag{5}$$

The control goal, that will be addressed for this application model in the following two subsections, is to guarantee asymptotic stability of the origin of the closed-loop consisting of (3), (4), and (5) for all transmission sequences $\theta^i := (\theta_k^i)_{k \in \mathbb{N}}$ that can result from the presented network model, i.e., from the constraints (1) and (2) together with a possible QoS guarantee for opportunistic transmissions for all $i \in \{1, \ldots, N\}$.

B. Stability Analysis and Synthesis Without Any Qos Assumptions on Opportunistic Traffic

To further analyze the behavior of the NCS, we introduce the sequence of successful transmission times for each NCS i as $\tau^i := (\tau_l^i)_{l \in \mathbb{N}}$, taking values in a subset of $\mathbb{N}_0$. Formally, the elements of the sequence are defined by the equivalence

$$k \in \tau^i \Leftrightarrow \theta_k^i = 1, \quad \forall k \in \mathbb{N}_0. \tag{6}$$

This subsection covers the case that no QoS guarantees are assumed for the opportunistic traffic at all. Thus, the only guarantee is provided by the deterministic transmissions, i.e., by (1) and (2). This guarantee implies that

$$\begin{aligned} \tau_1^i &\le s^i < d^i, \\ \tau_{l+1}^i - \tau_l^i &\le d^i, \forall l \in \mathbb{N}. \end{aligned} \tag{7}$$

Using this information, the transmissions of NCS i can be modeled as a network with bounded packet loss (cf. Definition 2 and Equation (2) in [11]), i.e., with a packet loss sequence

$$\eta^i := \left(\eta_{\tau_l^i}^i\right)_{\tau_l^i \in \tau^i}, \text{ with } \eta_{\tau_l^i}^i := \tau_{l+1}^i - \tau_l^i, \quad \forall \tau_l^i \in \tau^i, \tag{8}$$

which takes values in the set

$$\mathcal{F}^i := \{1, \ldots, d^i\} \tag{9}$$

arbitrarily.

We are now able to provide a sufficient condition for stability and a stabilization result.

Corollary 1: The origin of NCS i, consisting of (3), (4), and (5), is asymptotically stable for all sequences θ^i that can occur according to the slotted transmission classification model with guaranteed deterministic transmissions (1) and (2), if there exist matrices $P_g^i \succ 0$ for $g \in \mathcal{F}^i$, $Q^i \succ 0$, $Z^i \succ 0$, $N_1^i \in \mathbb{R}^{n^i \times n^i}$ and $N_2^i \in \mathbb{R}^{n^i \times n^i}$ such that

$$\begin{bmatrix} \Xi_{gh1}^i & \Xi_{gh2}^i & N_1^i \\ * & \Xi_{gh3}^i & N_2^i \\ * & * & -Z^i \end{bmatrix} \prec 0 \tag{10}$$

holds for all $g, h \in \{1, \ldots, d^i\}$, where

$$\begin{aligned} \Xi_{gh1}^i &= (R_h^i)^\intercal P_h^i R_h^i - P_g^i + Q^i + ((R_h^i)^\intercal - I) Z^i (R_h^i - I) \\ &\quad + N_1^i + (N_1^i)^\intercal, \\ \Xi_{gh2}^i &= (R_h^i)^\intercal P_h^i (A^i)^{h-1} B^i K^i \\ &\quad + ((R_h^i)^\intercal - I) Z^i (A^i)^{h-1} B^i K^i - N_1^i + (N_2^i)^\intercal, \\ \Xi_{gh3}^i &= ((A^i)^{h-1} B^i K^i)^\intercal (P_h^i + Z^i)(A^i)^{h-1} B^i K^i \\ &\quad - Q^i - N_2^i - (N_2^i)^\intercal, \\ R_h^i &= (A^i)^h + \sum_{r=0}^{h-2} (A^i)^r B^i K^i. \end{aligned} \tag{11}$$

Proof: As described above, the NCS i, combined of (3), (4), and (5), with (1) and (2) being the only restrictions on θ^i, can be modeled as a discrete-time system with a network-induced delay that is equal to the sampling time and a packet loss process that takes values in $\mathcal{F}^i$ arbitrarily. Thus, Theorem 16 in [11] directly proves the result. ■

Corollary 2: There exists a static state feedback controller (4) such that the origin of NCS i, combined of (3), (4), and (5), is asymptotically stable for all sequences θ^i that can occur according to the slotted transmission classification model with guaranteed deterministic transmissions (1) and (2), if there exist matrices $X_g^i \succ 0$ for $g \in \mathcal{F}^i$, $\bar{Q}^i \succ 0$, $W^i \succ 0$, $G^i \in \mathbb{R}^{n^i \times n^i}$, $\bar{N}_1^i \in \mathbb{R}^{n^i \times n^i}$, $\bar{N}_2^i \in \mathbb{R}^{n^i \times n^i}$, and $Y^i \in \mathbb{R}^{p^i \times n^i}$ such that

$$\begin{bmatrix} \Upsilon_{1g}^i & * & * & * & * \\ -(\bar{N}_1^i)^\intercal + \bar{N}_2^i & \Upsilon_2^i & * & * & * \\ (\bar{N}_1^i)^\intercal & (\bar{N}_2^i)^\intercal & \Upsilon_3^i & * & * \\ \Gamma_h^i - G^i & (A^i)^{h-1} B^i Y^i & 0 & -W^i & * \\ \Gamma_h^i & (A^i)^{h-1} B^i Y^i & 0 & 0 & -X_h^i \end{bmatrix} \prec 0 \tag{12}$$

for all $g, h \in \{1, \ldots, d^i\}$, where

$$\Upsilon^i_{1g} = X^i_g - G^i - (G^i)^\intercal + \bar{Q}^i + \bar{N}^i_1 + (\bar{N}^i_1)^\intercal,$$
$$\Upsilon^i_2 = -\bar{Q}^i - \bar{N}^i_2 - (\bar{N}^i_2)^\intercal,$$
$$\Upsilon^i_3 = W^i - G^i - (G^i)^\intercal,$$
$$\Gamma^i_h = (A^i)^h G^i + \sum_{r=0}^{h-2} (A^i)^r B^i Y^i. \qquad (13)$$

In this case, the controller gain can be computed as $K^i = Y^i(G^i)^{-1}$.

Proof: The result follows directly from Theorem 17 in [11]. ■

Remark 3: Modeling the case at hand, i.e., that only deterministic transmissions are guaranteed and no knowledge about opportunistic transmissions is available, as a network with bounded packet loss is no equivalent modeling. The reason for this is that the knowledge about fixed deterministic transmissions carries more information than solely boundedness of packet loss. An alternative modeling that is indeed equivalent, comes with a slightly more complicated approach as a special case of the results in the next subsection.

C. Stability Analysis and Synthesis with Partial QoS Guarantees on Opportunistic Transmissions

This subsection is concerned with the question how one can take potential QoS information about opportunistic transmission slots into account. The goal is to modify the stability analysis and synthesis results from the previous subsection such that some additionally available QoS information can be exploited. As one exemplary type of QoS specifications for opportunistic transmission slots, we consider the case where one knows that between any two consecutive deterministic transmission slots of NCS i, at least $m^i \in \mathbb{N}_0$, ($m^i \leq d^i - 1$), opportunistic transmissions are successful. This means that, out of the $d^i - 1$ opportunistically transmitted datagrams in such an interval, at least m^i are received within the delay bound T_s. However, in this type of QoS specification, one does not assume a priori knowledge about which opportunistic transmission is successful. Furthermore, it is still possible that more than m^i opportunistic transmissions are successful between two deterministic transmissions. This guarantee can again be formulated as a constraint on θ^i as

$$\left(\sum_{k=s^i+jd^i+1}^{s^i+(j+1)d^i-1} \theta^i_k \right) \geq m^i, \quad \forall j \in \mathbb{N}_0. \qquad (14)$$

Such a QoS specification is similar, although not equal, to the concept of (m,k)-firmness [12], which is a subclass of weakly hard real-time (WHRT) constraints [13]. Thus, we intend to extend the result from the previous subsection with a similar approach as in [14] (for NCS with (m,k)-firm network descriptions), and [15] and [4] (for NCS with WHRT network descriptions). Those works use a directed graph to describe and exploit the restriction of the possible sequences θ^i that is imposed by the additional QoS information.

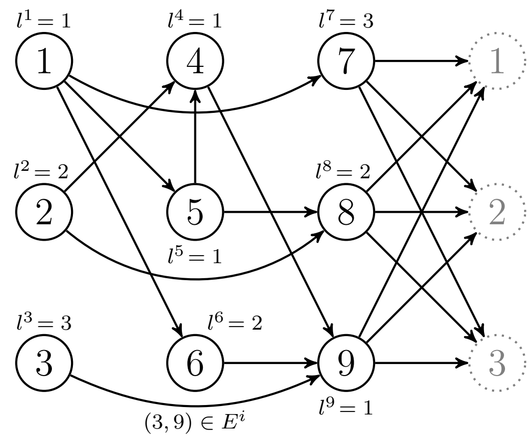

Fig. 3. Example for a QoS graph $\mathcal{G}^i$ for a slotted transmission classification model with $d^i = 4$ and $m^i = 1$, where nodes 1, 2, and 3 are depicted twice to simplify presentation.

A directed graph, or digraph, with labeled vertices, $\mathcal{G} = (V, E, L, l)$, consists of a set of vertices V, a set of edges E, a set of vertex labels L, and an injective function $l : V \to L$ that assigns a label to every vertex in V. The elements of E are ordered pairs of vertices (v_g, v_h) where v_g is the head and v_h is the tail of the edge. Two vertices v_h and v_g are said to be adjacent if $(v_g, v_h) \in E$. Using these definitions, we can now define the concept of QoS graphs that will be used to state the stability results.

Definition 4: A digraph with labeled vertices $\mathcal{G}^i$ is said to be a *QoS graph* for a slotted transmission classification model with a period of deterministic slots d^i and at least m^i successful transmissions between every two deterministic transmissions, i.e., for a model in which the transmission sequences θ^i are restricted by (1), (2), and (14), if for all possible resulting packet loss sequences η^i, as defined in (8), every two successive elements in the sequence η^i are labels of adjacent vertices of the graph $\mathcal{G}^i$.

An example for a QoS graph for the case $d^i = 4$ and $m^i = 1$ is given by Fig. 3. Before we use this definition for stating the stability analysis result, we give an example how transmission and loss sequences are generated by the graph in Fig. 3.

Example 5: As an example, we consider a network with $d^i = 4$, $m^i = 1$, i.e., the case of Fig. 3, and $s^i = 0$. For this case, a possible transmission sequence θ^i that captures whether transmissions are successful or not, is given by $\theta^i = (\mathbf{1}001\mathbf{1}110\mathbf{1}\cdots)$, where deterministic transmissions are boldfaced. The corresponding sequence τ^i that captures the time instants of successful transmissions, is given by $\tau^i = (034568\cdots)$ and accordingly the loss sequence η^i that captures the time between two successful transmissions, is given by $\eta^i = (31112\cdots)$. According to Definition 4, it must be possible to represent every two successive elements in the sequence η^i by labels of adjacent vertices of the graph $\mathcal{G}^i$ in Fig. 3. This is possible with the sequence of adjacent vertices $(39158\cdots)$.

Corollary 6: The origin of NCS i, combined of (3), (4), and (5), is asymptotically stable for all sequences θ^i that can occur according to the slotted transmission classification model with guaranteed deterministic transmissions (1), (2) and partial guarantees for opportunistic transmissions (14), if there exist matrices $P_g^i \succ 0$ for $g \in V^i$, $Q^i \succ 0$, $Z^i \succ 0$, $N_1^i \in \mathbb{R}^{n^i \times n^i}$ and $N_2^i \in \mathbb{R}^{n^i \times n^i}$ such that

$$\begin{bmatrix} \tilde{\Xi}_{gh1}^i & \tilde{\Xi}_{gh2}^i & N_1^i \\ * & \tilde{\Xi}_{gh3}^i & N_2^i \\ * & * & -Z^i \end{bmatrix} \prec 0 \tag{15}$$

holds for all g, h s.t. $(g, h) \in E^i$, where $\mathcal{G}^i = (V^i, E^i, L^i, l^i)$ is the corresponding QoS graph and

$$\begin{aligned} \tilde{\Xi}_{gh1}^i &= (\tilde{R}_h^i)^\intercal P_h^i \tilde{R}_h^i - P_g^i + Q^i + ((\tilde{R}_h^i)^\intercal - I) Z^i (\tilde{R}_h^i - I) \\ &\quad + N_1^i + (N_1^i)^\intercal, \\ \tilde{\Xi}_{gh2}^i &= (\tilde{R}_h^i)^\intercal P_h^i (A^i)^{l^i(h)-1} B^i K^i \\ &\quad + ((\tilde{R}_h^i)^\intercal - I) Z^i (A^i)^{l^i(h)-1} B^i K^i - N_1^i + (N_2^i)^\intercal, \\ \tilde{\Xi}_{gh3}^i &= ((A^i)^{l^i(h)-1} B^i K^i)^\intercal (P_h^i + Z^i)(A^i)^{l^i(h)-1} B^i K^i \\ &\quad - Q^i - N_2^i - (N_2^i)^\intercal, \\ \tilde{R}_h^i &= (A^i)^{l^i(h)} + \sum_{r=0}^{l^i(h)-2} (A^i)^r B^i K^i. \end{aligned} \tag{16}$$

Proof: The result can be proven by a combination of the proof of Theorem 16 in [11] with the fact that the set of sequences of labeled vertices that can be generated by the QoS graph produces all possible loss sequences η^i as defined in Definition 4. ■

The stabilization result of the previous subsection can be modified for the QoS specification in this subsection with the same approach. The result is omitted here due to space limitations.

Remark 7: The case without any QoS guarantees at all is covered by the special case $m^i = 0$ within this framework.

Remark 8: The complexity of the results in the previous subsection and this subsection can be assessed by comparing the number of evaluations of LMI (10) and (15). The corresponding LMI has to be evaluated $\sum_{i=1}^N (d^i)^2$ times for Corollary 1 and $\sum_{i=1}^N |E^i|$ times for Corollary 6.

V. Conclusion and Outlook

In this paper, we presented a communication model for NCS based on a time-slotted classification of deterministic and opportunistic transmissions. By illustrating the feasibility of implementing a corresponding communication service using TSN, and providing tools for stability analysis with no or only limited assumptions about opportunistic transmissions, we demonstrated that this model serves as a fairly generic interface between control systems and communication networks for analysis and design purposes.

Moreover, there are several interesting questions in communication networks, in control, and concerning the integration of communication and control that arise from the presented communication model. How, for instance, can network resources for *opportunistic* transmissions be shared by multiple NCS such that their performance is maximized? Possible approaches include packet priority based scheduling [16] in the network or the use of performance-oriented event-based control methods [17] in the control system. To this end, we plan to extend previous work to explicitly consider deterministic transmissions in the scheduling or event-triggering decisions for opportunistic transmissions. Furthermore, if time-varying controllers are used in opportunistic slots to greedily optimize performance, the results of this paper need to be modified accordingly such that stability guarantees can be preserved.

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